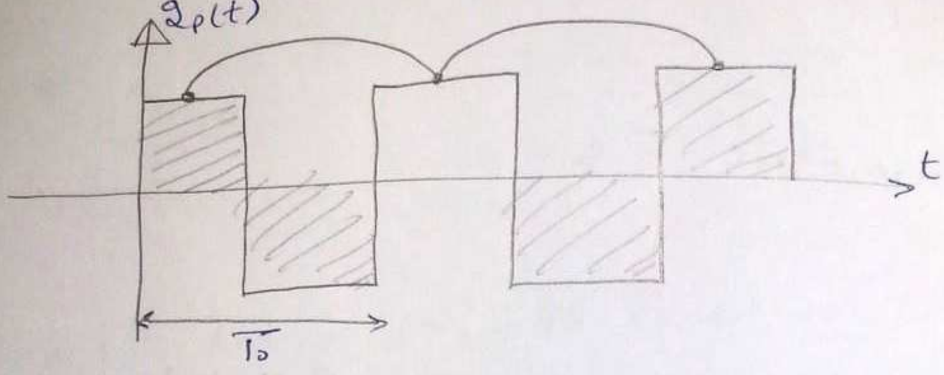
د/مرام سكشن (1) (اتصالات)

٢٠١٠/٩/١٠

Fourier Series

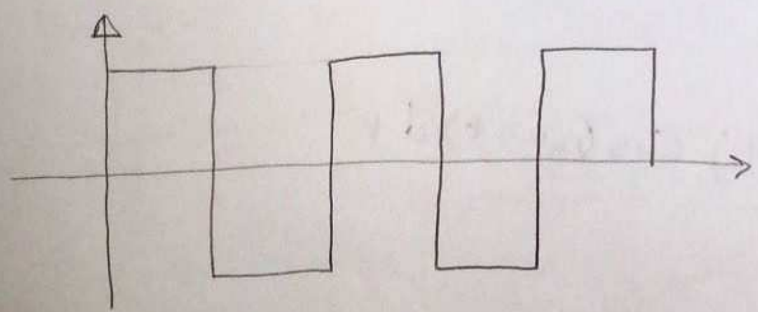
* F.s used to expand periodic signals into an infinite sum of series & Cosines.



→ for periodic signals

$g_p(t)(t \pm mT_0) = g_p(t)$, m integer.

→ for the previous wave :-



* There is
① DC → a_0
② infinite sines
③ " C-sines

f_0 → fundemental freq. $= \frac{1}{T_0}$

← يوجد عدد لا نهائي من sines و cosines بأخذ الاعتبار الـ f_0

Sines → $(f_0, 2f_0, \ldots \infty)$

$$g_p(t) = a_0 + 2\sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + 2\sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$\rightarrow$ Trignometric F.s

$\rightarrow$ Coeff:-

$$* \; a_0 = \frac{1}{T_0}\int_0^{T_0} g_P(t)\, dt \quad \rightarrow \text{avg. of } g_P(t)$$

$$= \frac{\text{Area under } g_P(t) \text{ on interval of } T_0}{T_0}$$

$$* \; a_n = \frac{1}{T_0}\int^{T_0} g_P(t) \cos(n\omega_0 t)\, dt$$

$$* \; b_n = \frac{1}{T_0}\int^{T_0} g_P(t) \sin(n\omega_0 t)\, dt$$

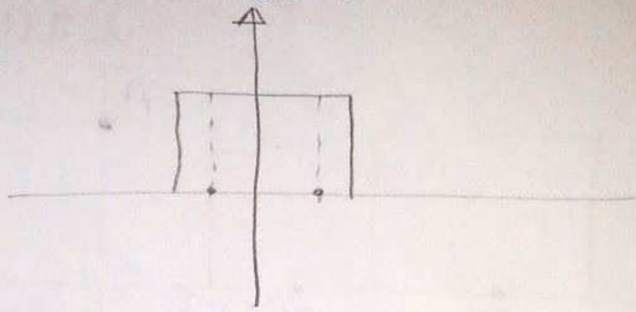
Notes

* if $g_p(t)$ is even Fn.

$g_p(-t) = g(t)$

$\therefore b_n = 0$

a_0, b_n ✓

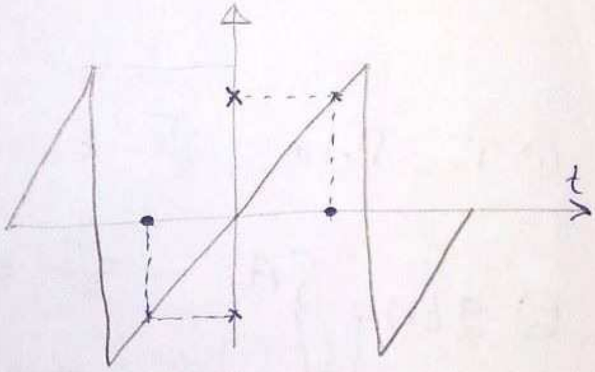


* if $g_p(t)$ is odd function

$g_p(-t) = -g_p(t)$

$\therefore a_0 = a_n = 0$

b_n ✓



خطوات حل أي مسألة

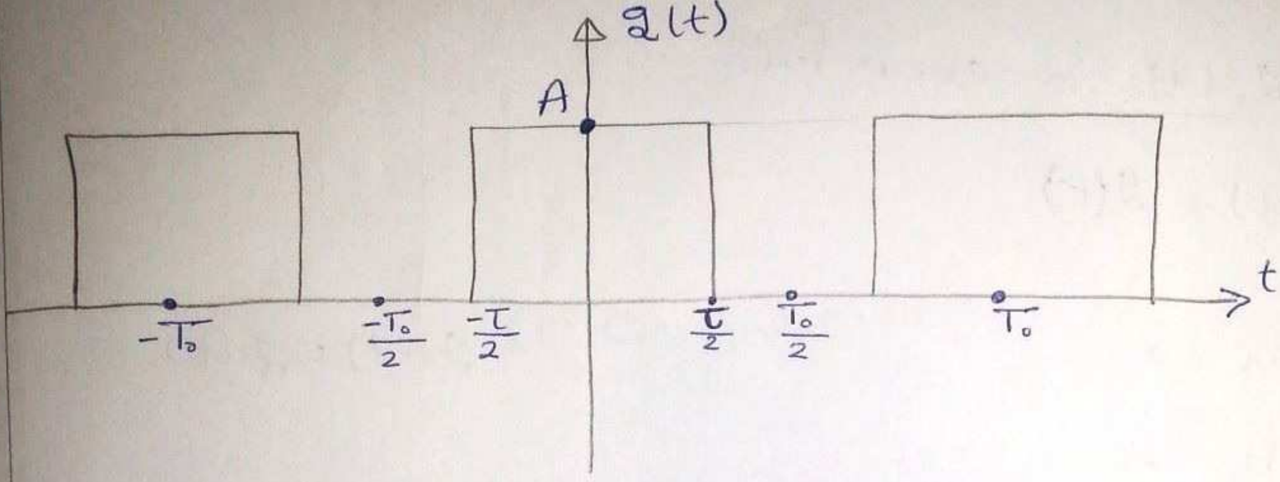
١- نحدد T_0.

٢- نحدد معادلة الدالة خلال T_0.

٣- "سؤال" هل الدالة زوجية أم فردية

٤- نحسب a_0 , a_n , b_n ونعوض في معادلة (Fourier series).

Ex : For the periodic Pulse train. Find F.s



$\tau \longrightarrow$ Pulse width

$\frac{-T_0}{2} \longrightarrow \frac{T_0}{2}$ نأخذ الفترة ←

① T_0 from $\frac{-T_0}{2}$ to $\frac{T_0}{2}$

② $g(t) = \begin{cases} A & \frac{-\tau}{2} \leq t \leq \frac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}$

③ $g(t)$ is even fn. $\Rightarrow b_n = 0$

④ $a_0 = \frac{1}{T_0} \int^{T_0} g_p(t)\, dt$

$= \frac{1}{T_0} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A \cdot dt$

$$a_0 = \frac{A}{T_0} \left| t \right|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \frac{A\tau}{T_0}$$

$$\boxed{a_0 = \frac{A\tau}{T_0}}$$

$$a_n = \frac{1}{T_0} \int^{T_0} gp(t) \cos(n\omega_0 t)\, dt$$

$$= \frac{1}{T_0} \int_{-\frac{\tau}{2}}^{\tau/2} A \cdot \cos(n\omega_0 t)\, dt$$

$$a_n = \frac{A}{n\omega_0 T_0} \left| \sin(n\omega_0 t) \right|_{-\frac{\tau}{2}}^{\frac{\tau}{2}}$$

$$= \frac{A}{n \cancel{T_0} \frac{2\pi}{\cancel{T_0}}} \left[\sin\left(n\omega_0 \frac{\tau}{2}\right) - \sin\left(n\omega_0 \left(\frac{-\tau}{2}\right)\right) \right]$$

$$a_n = \frac{A}{2n\pi} \cdot 2 \sin\left(n\omega_0 \frac{\tau}{2}\right)$$

$$\boxed{a_n = \frac{A}{n\pi} \sin\left(\frac{n\pi\tau}{T_0}\right)}$$

$$\therefore g_p(t) = a_0 + 2\sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + 2\sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$$\boxed{g_p(t) = \frac{A\tau}{T_0} + 2\sum_{n=1}^{\infty} \frac{A}{n\pi} \sin\left(\frac{n\pi\tau}{T_0}\right) * \cos(n\omega_0 t)}$$

بعد اجراء بعض الاختصارات في المعادلة السابقة أو في القانون السابق.

Complex F.S

$$g_p(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{+Jn\omega_0 t}$$

$$C_n = \frac{1}{T_0} \int^{T_0} g_p(t) \cdot e^{-Jn\omega_0 t}$$

← هذه الطريقة تستخدم لرسم (Spectrum).

← الإشارة مع ال (frequency) هي (Spectrum)

$$C_n = |C_n| \cdot e^{J\theta_n}$$

$|C_n| = \sqrt{a_n^2 + b_n^2}$

$|C_0| = \sqrt{a_0^2 + \cancel{b_0^2}}_{\to 0} \Rightarrow \boxed{|C_0| = a_0}$

Power in F.S

* For any periodic signal $g_p(t)$, the avg. Power content is given by:

$$P_{avg} = \frac{1}{T_0} \int^{T_0} g_p^2(t)\, dt$$

or

$$P_{avg.} = \sum_{n=1}^{\infty} |C_n|^2$$